ANALYSIS OF INNOVATION: A GAME THEORETIC MODEL

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Abstract

This paper attempts to model the impact of innovation and imitation in a duopoly market on firm profitability. The market is initially taken to be a duopoly because of the relative ease in modelling and it also provides the minimum situation required for any sort of competitive analysis. This paper focuses specifically on product innovation and not process innovation and so innovation has an impact on the demand. First, we model the demand curves and the cost curves of the individual firms in a manner that facilitates our analysis. Second, we find the short run and the long run profits of both firms. Here, the difference between the short run and the long run is defined as the time lag that it takes for the imitating firm to imitate the innovator's product to some degree. The short run analysis is a one-stage game involving price competition. The long run analysis is a two-stage game where stage one is determining the degree of imitation and stage two determines the price in a similar manner to the short-term analysis. A positive spillover of innovation is observed both in the short and in the long run. Third, we also analyse the impact of various parameters such as price elasticity, degree of imitation and the market's affinity to new products on the incentive to innovate, thus, providing important insights into which markets are characterised by more product innovation and why some industries demand more protection. Fourth, we look at how the positive spillover effect may be tackled from a policymaker's perspective. Finally, the paper attempts to provide an intuitive explanation of how this model can be extended to 'n' firms.

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1. INTRODUCTION

Competition among a small number of firms has always attracted much interest from theoretical economists. Price and quantity competition has been much easier to comprehend and model, as done in the Cournot, Bertrand and Stackelberg models. Analysis of competition in other spaces has been much more elusive but equally as important, especially among a small number of firms. The influential paper by Harold Hotelling in the first half of the 20th century was one of the first in an increasingly large number of attempts to model various intricacies of competition involved. We try to model one such important element of competition and its impact on the short term and the long-term profitability of the firms involved in such competition. That element is innovation and more specifically product innovation. The inspiration of this paper in its analysis of innovation follows directly from the work done by growth economist Paul Romer in his "Increasing Returns and Long-Run Growth", which makes it clear that technological progress is the engine of long-term growth. The product innovation analysed in this paper is clearly not perfectly synonymous with the macroeconomic concept of technological progress. However, the importance of the growth of ideas or the increase in the output of the 'research sector' as emphasised

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by Romer has clear parallels with the analysis carried out in this paper.

There are essentially two types of innovation, product and process innovation. Product innovation involves a change in the attribute of the product while process innovation involves improvement in the process of producing a given product. Process innovation usually has implications for the cost of production and is easier to model as a result. Product innovation, on the other hand, has implications for demand, market share as well as the mark-up of price over cost and hence the profitability. An important insight into process innovation as well as the treatment of cost of imitation comes from Rosenberg and Landau in their influential book, "The Positive Sum Strategy: Harnessing Technology for Economic Growth" which talks about lower imitation costs and process innovation's impact on the supply curve.

The firms competing over the same market often gain on other firms' loss, so innovation has implications not only for the firm engaging in innovation but also for the competitors, which might induce imitation. In fact, the fear of imitation has been the primary reason for patent and protection policies around the world, to prevent the spillover of benefits (positive externalities) of innovation to other firms in order to promote innovation in the first place.

The model examined in this paper focuses on a two-firm situation. The model is deliberately kept simplistic, where one firm gains market only to the detriment of the other firm with total market demand remaining the same. The demand curves utilised in this paper are inspired by the work done by P. Krugman in his analysis of monopolistically competitive firms in his book on international economics, with the equation adjusted to meet the specific needs of our model. The implications of innovation are derived in both the short term (when the other firm does not act) and the long-term (when the other firm has the opportunity to imitate). The focus is on the degree of imitation i.e. how closely the second firm copies the first firm's product. The cost of innovation is modelled through a basic linear cost function, where the degree of imitation correspondingly affects the cost of the imitating firm. The competition in terms of price is similar to the traditional Bertrand competition. Finally, policy suggestions are presented based on the policymaker's objective for promoting more innovation and competition, given the situation of an existence of a positive spillover effect of innovation which cannot be effectively internalised by the innovating firm without State intervention in the market. This paper attempts to arrive at a conclusion regarding which policy will work effectively within the framework of the model.

2. MODEL

2.1. Assumptions

The model analysed in this paper consists of only 2 firms, where firm 1 is taken as the innovator and firm 2 as the imitator. The process of entry and exit is not considered in our model as this would lead us to analyse existing experience/initial advantage of any firm in the industry (Krugman, 2015). We ignore any cost of establishment i.e. fixed cost, which is in line with the assumption of no entry and exit. We also assume that any sort of innovation leads to horizontal differentiation and not vertical. Horizontal differentiation means that some consumers prefer one good over the other and some prefer the latter over the former if offered at the same price while vertical differentiation means when one good is preferred over the other universally if offered at the same price.

Both firms have similar marginal costs (m). This is again done for simplicity. The model can be subsequently extended to incorporate more complex cost functions. The cost of innovation is taken to be a constant fixed investment (I) since we are taking the case of product innovation which involves invention of new attributes in the product which requires an investment in R&D. Imitation cost rises linearly with the degree of imitation, this is represented by an increase in $F\gamma$. Thus, close imitation in this model increases the cost for the imitator but this increase in cost is always assumed to be lower than the initial sunk cost incurred by the innovator i.e. $F\gamma < I$. We will analyse the effects in both the short run and the long run, where we define the short run as the period when the imitator cannot react to the innovation and the long run as the period when the other firm can imitate the innovator's product to some degree.

The cost structure of the innovator (taken to be firm 1) stays the same in both the short run and the long run —

$$C_1 = \begin{cases} I, q_1 = 0 \\ I + mq_1, \ q_1 > 0 \end{cases} \tag{1}$$

I : investment in R&D (sunk cost)
m : marginal cost (some positive constant)
q_1 : quantity produced by the innovator

The cost structure of the imitator (taken as firm 2) changes when we move from the short run to the long run in the following manner —

Imitator's short run cost:

$$C_2 = \begin{cases} 0, q_2 = 0 \\ mq_2, \ q_2 > 0 \end{cases} \tag{2}$$

Imitator's long run cost:

$$C_2 = \begin{cases} F\gamma, q_2 = 0 \\ F\gamma + mq_2,\ q_2 > 0 \end{cases} \quad ; \gamma \in [0,1], F\gamma < I \tag{3}$$

F: linear cost multiplier (some positive constant)
γ : coefficient of imitation
q_2 : quantity produced by the imitator
$F\gamma$: linear cost of imitation

2.1.1. Coefficient of Imitation (γ)

The coefficient of imitation reflects the degree of imitation by firm 2, where $\gamma = 0$ signifies that firm 2 does not imitate the product of firm 1 at all and $\gamma = 1$ analogously refers to the situation when firm 2 fully imitates firm 1's product. When the coefficient of imitation lies between 0 and 1 i.e. $\gamma \in (0,1)$, then firm 2 copies some of the attributes of firm 1's product while maintaining some differentiation of its own. Imitation also involves an additional cost given by $F\gamma$, where F is the linear cost multiplier. When $\gamma = 0$, there is no cost of imitation, when $\gamma = 1$, the cost of imitation becomes equal to F .

2.1.2. Demand Equations

$$q_1 = \frac{A}{2} - \frac{(p_1 - p_2)b}{(1-\gamma)} + c(1-\gamma)I \tag{4}$$

$$q_2 = \frac{A}{2} + \frac{(p_1 - p_2)b}{(1-\gamma)} - c(1-\gamma)I \tag{5}$$

A: total market size
p_1: price of firm 1
p_2: price of firm 2
$\frac{b}{(1-\gamma)}$: responsiveness of the quantity demanded to the price differential i.e. $(p_1 - p_2)$
$c(1-\gamma)$: responsiveness of the quantity demanded to the investment expenditure i.e. I

The demand curves are taken such that whatever prices and the other parameters may be, the total market size remains equal to A. This means that in theory, both the firms can get infinite profits if they maintain an adequate price differential while subsequently increasing the absolute value of the prices as much as they wish i.e. we take the market to be perfectly inelastic in prices. However, in reality, the market would not be perfectly inelastic and the firms would have to charge accordingly. The reason for the same is that firstly, we assume that the firms cannot collude, their own price is constrained by what is charged by the other firm and secondly, we want to analyse the impact of innovation and imitation decisions by the firms and not the price decisions which have already been extensively modelled before.

The idea behind the coefficient of imitation appearing in the demand curves is analogous to theory, so an innovation in a product should impact consumer choices and thus the demand curve in the same way that an innovation in process impacts the cost of production and subsequently the supply curve (Rosenberg 1986). How the coefficient of imitation in the equations taken here affects demand can be appreciated when we try and understand what the demand curves will look like under different situations. Suppose, both the firms have the same product and the consumers cannot differentiate between them then γ will be equal to 1 and the coefficients of the demand curves will be transformed such that if any firm charges a higher price than the other, it loses all its market share and the other firm captures the entire market. The only way both firms get any market share is if both the firms charge the same price and their market share will be given by $\frac{A}{2}$ each. This situation is summarised below along the lines of the general Bertrand duopoly result with highly substitutable products (Bertrand, 1883).

$$q_i = \begin{cases} A & , p_i < p_j \\ 0 & , p_i > p_j \\ \frac{A}{2} & , p_i = p_j \end{cases}$$

Subsequently, if γ falls from 1 both the coefficients will be finite and there will be some scope of price differences between the 2 firms at profit-maximising equilibrium.

2.2. Short Run Analysis

2.2.1. Before Innovation

Before either firm takes on any type of innovative activity both the firms virtually sell the same product. Since $I = 0$, both the firms only incur marginal cost of production which in turn depends on the quantity that they produce. Since there is no innovation and hence no product differentiation the coefficients will be such that they cannot sell at a price different to the other, simple Bertrand competition implies that the price of each firm will be driven down to their marginal costs which are taken as 'm' in our model. Subsequently, each firm will earn zero profit in the equilibrium. This entire situation can be summarised as below:

$$p_1^* = p_2^* = m \ ; \ I = 0 \ ; \ q_1^* = q_2^* = \frac{A}{2} \ ; \ \pi_1^* = \pi_2^* = 0$$

2.2.2. After Innovation

If firm 1 decides to innovate in the short run, given the assumption that in the short run firm 2 cannot imitate firm 1's product, the coefficient of imitation will be equal to zero i.e. $\gamma = 0$. This will lead to the following short run demand curves:

$$q_1 = \frac{A}{2} - (p_1 - p_2)b + cI \quad (6)$$

$$q_2 = \frac{A}{2} + (p_1 - p_2)b - cI \quad (7)$$

The cost structure in the short run is given by (1) for firm 1 and (2) for firm 2. Subsequently, by finding out the Nash equilibrium in price competition (algebra involved in solving for equilibrium is left for Appendix 1), the prices for each firm and the price differential are found as:

$$p_1^* = \frac{6mb + 3A + 2cI}{6b} \quad (8)$$

$$p_2^* = \frac{6mb + 3A - 2cI}{6b} \quad (9)$$

$$p_1^* - p_2^* = \frac{2cI}{3b} \quad (10)$$

The subsequent equilibrium profits are:

$$\pi_1^* = \frac{4c^2I^2 + 12cAI - 36bI + 9A^2}{36b} \quad (11)$$

$$\pi_2^* = \frac{-20c^2I^2 + 24AcI + 72cbIm + 9A^2}{36b} \quad (12)$$

2.3. Long Run Analysis

In the long run, firm 2 is assumed to have the ability to carry out some degree of imitation. The long run analysis is done through a two-stage decision-making process. In the first stage, firm 2 will decide on the degree of imitation i.e. γ and the second stage will involve a price competition between the firms. The analysis is done through backward induction where the Bertrand equilibrium prices (the second stage) are found first and then we look at how firm 2 decides on a coefficient of imitation that will allow it to maximise profit found in the second stage. The demand curves in this situation are given by (4) and (5) and the cost functions by (1) and (3). The Bertrand equilibrium prices will come out to be —

$$p_1^* = \frac{6mb + 3A(1-\gamma) + 2cI(1-\gamma)^2}{6b} \quad (13)$$

$$p_2^* = \frac{6mb + 3A(1-\gamma) - 2cI(1-\gamma)^2}{6b} \quad (14)$$

and the subsequent equilibrium profits are:

$$\pi_1^* = \frac{4c^2I^2(1-\gamma)^3 + 12cAcI(1-\gamma)^2 + 9A^2(1-\gamma) - 36bI}{36b} \quad (15)$$

$$\pi_2^* = \frac{-20c^2I^2(1-\gamma)^3 + 24AcI(1-\gamma)^2 + 72cbIm(1-\gamma) + 9A^2(1-\gamma)}{36b} - F\gamma \quad (16)$$

where $\gamma \in (0,1)$. At $\gamma = 0$, profits are same as equations (11) & (12) and at $\gamma = 1$, innovator profit is zero and imitator's is $-F\gamma$, as it is similar to the Bertrand competition when there is no differentiation in the product. The discussion on how an optimum degree of imitation is chosen by the imitator is carried out in section 3.2.

3. INTERPRETATIONS

3.1. Short Run

By looking at the short run equilibrium results, we can see that a price differential has emerged. This tells us that the profits of each firm will be at least as much as it was before innovation i.e. 0. To see why this is the case, see that the imitator can take 0 profit if he produces zero output as his cost function remains the same as before, so if the maximum profit he gets by producing a positive output is negative then he can earn a zero profit by not producing anything in the short run. Only if its profit is greater than zero when he produces a positive output will it have an incentive to produce an output greater than zero. If we look at the innovator, since its cost function has changed after incurring the innovation cost I, firm 1 can have a negative profit but since there's perfect information and if the prices are such that the negative profit becomes a reality, he will not undertake investment in the first place.

In the short run, innovation by one firm has a positive spillover effect for the other, i.e. it benefits not only the innovating firm but also the competing firm.

Equation (10) also makes it clear that firm 1's price is greater than that of firm 2 after innovation. This price differential is also a positive function of c, I and an inverse function of b.

Higher investment and higher investment responsiveness of the market give firm 1 the ability to set a higher mark-up over firm 2's price, while a greater price elasticity of demand diminishes this ability.

3.2. Long Run

The effect of c, b and γ on π_1^* and the effect of γ on π_2^* is formally assessed by utilising the method of partial derivatives in Appendix 2.

If we look at the equilibrium profit of firm 1, the impact of 'b' is unambiguously negative which can be interpreted as the more elastic the demand to the price difference, the less is an incentive to innovate. To see why this is the case, say, all the other parameters are constant then the positive effect of innovation on the ability to charge a higher price than the other is diminished if 'b' is large.

Thus, the lesser the price elasticity of demand in the market, the *greater is the incentive to innovate. The impact of 'c ' is positive throughout which means that investment will always have an unambiguously positive impact on the profit.*

The linear cost of imitation i.e. $F\gamma$ acts as a deterrent to imitation. Any market where it is easy to imitate without incurring a substantial cost, there will naturally be a higher incentive to imitate and subsequently a lower incentive to innovate. This intuitive result is confirmed by our model in the following manner –

If the linear cost of imitation $F\gamma$ is low, firm 2 can gain a higher profit and has an incentive to imitate to a higher degree. A high γ, in turn, lowers the profit of firm 1 leaving it with a lesser incentive to innovate.

There are both positive and negative impacts of imitation on firm 2's equilibrium profit –

As firm 2 imitates more and makes its product increasingly similar to that of firm 1, it consequentially gains a higher market share by siphoning away some of the customers from firm 1. However, by carrying out this imitation firm 2 also incurs a higher cost i.e. a higher linear cost of imitation, $F\gamma$.

At the same time, firm 2 loses some of its independent price-setting ability as the products being sold by both firms become increasingly similar and price becomes an important determinant of consumer choice between these now similar products or in other words, price competition intensifies.

Given the long run second stage Nash equilibrium results, utilising backward induction firm 2 essentially solves the following maximisation problem –

$$\underset{\gamma \in [0,1]}{Max}\ \pi_2^* = \frac{-20c^2I^2(1-\gamma)^3 + 24AcI(1-\gamma)^2 + 72cbIm(1-\gamma) + 9A^2(1-\gamma)}{36b} - F\gamma$$

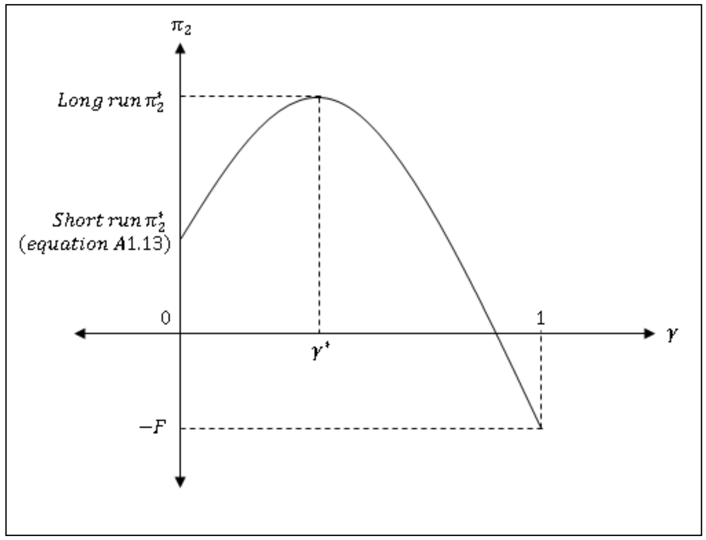
Firm 2 chooses γ to maximise his second stage optimal profit at each level of γ. To see how this is done we must appreciate that at $\gamma = 0$, the profit is positive as discussed above and at $\gamma = 1$, it is negative. So, it must go up as well as down in the interval [0,1]. Say it only goes down then it chooses to not imitate at all, i.e. $\gamma = 0$. It cannot only go up since it ends up below where it started. So, the only possible options are $\gamma \in [0,1)$ for the choice of the optimal. We look at one such scenario in Figure 1 where the maximum profit goes up and then down and the optimal maximum profit is achieved at γ^*.

The long run price differential comes out to be -

$$p_1^* - p_2^* = \frac{4cI(1-\gamma)^2}{6b} \tag{17}$$

Equation (17) signifies that in the long run, the price of firm 1 can be greater than or equal to that of firm 2. If

***Figure 1**: Sub-game perfect equilibrium profit of firm 2 (a possible illustration)*



$\gamma = 1$ i.e. firm 2 fully imitates firm 1, then the price differential disappears and the model reverts to the Bertrand Competition result with homogenous goods with prices driven down to the marginal cost i.e. $p_1^* = p_2^* = m$. However, this case can be ruled out since firm 2 is utilising backward induction to obtain a global maximum point $\gamma^* \in [0,1)$ shown in Figure 1.

The long run price differential shows that an increasing level of imitation corresponds to lesser and lesser control over prices for both the firms.

This explains the result in Appendix 2, equation (A2.3) –

A higher γ corresponds to lesser price control for firm 1 and thus a lesser incentive to innovate or in simple words, a higher expected level of imitation reduces the incentive to innovate for firm 1.

4. POLICY IMPLICATIONS

This section deals with the question of an 'optimal' policy from the point of view of a benevolent policymaker trying to maximise social welfare. It is important to note that any analysis of policy recommendations must include perspectives from both the consumption and the production side. This paper has so far only discussed the production perspective so as to arrive at a result of optimal profitability for the existing firms in the market. We continue to look at what leads to an 'optimal' policy by focusing majorly on the producers' side of the market while providing only a brief explanation of what might occur from the consumers' perspective as well. The discussion in this section remains preliminary in the sense that a rigorous analysis from the consumers' point of view might lead to results that contradict the policy recommendations on the basis of only a brief look at the consumers' side.

The policymaker's objective of maximising social welfare might translate into the promotion of innovation or encouraging more competition in the market. It is often argued that some protection must be given to the innovator at the expense of perfect competition to promote the advancement of innovative activity. We consider both of these objectives. As we have seen, there is a positive spillover effect in the model persistent both in the short and long run. This means that given 'commodity', i.e., innovation will be under-produced since firm 1 cannot internalise this positive externality.

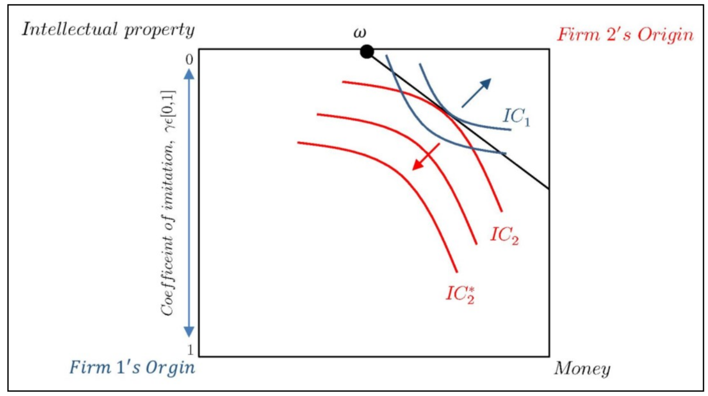
4.1. Patent Policy

A patent policy will provide well-defined property rights of the innovation (intellectual property rights) to the innovator, such that he can trade such rights with the other firms who wish to purchase that product technology. The innovator will subsequently internalise this profit as the sale price will be in equilibrium such that it is Pareto efficient (Coase, 1960).

With reference to Figure 2 which looks at a hypothetical Edgeworth box illustration, if we assume Cobb-Douglas utility function for both the firms, we get well-shaped utility level curves as shown in the diagram. The preference direction of firm 1's indifference curves should be clear from the result in Appendix 2 equation A2.3. The preference direction for firm 2 can be directly seen in Figure 1. As γ increases, π_2 increases as well but π_2 reaches a maxima at $\gamma = \gamma^*$ after which an increase in γ leads to a fall in π_2, this means that there will be a bliss point for firm 2 at γ^* and correspondingly the highest possible IC given IC_2^* in Figure 2 beyond which 'intellectual property rights' essentially become a 'bad' commodity for firm 2. If well-defined intellectual property rights are given to firm 1, then the initial point where both firms operate is at ω, where the coefficient of imitation is zero i.e. firm 2 cannot legally imitate firm 1's product. Firm 1 might decide to stay at ω or it can trade its intellectual property rights by allowing firm 2 to imitate its product to a certain degree in return for compensation in terms of money, once again leading to internalisation of the positive spillover effect of innovation.

The enforcement of such a patent policy would imply that firm 1 will remain as the sole producer/seller of its innovated product in the market. A situation such as this would generally lead to the establishment of a monopoly and a corresponding loss of the entire consumer surplus due to monopoly pricing. However, given the assumption of horizontal differentiation in this model, consumers do not objectively prefer one good over the other, i.e., the innovated product of firm 1 is not preferred universally over firm 2's product by the consumers. Thus, firm 2 will still retain market share rather than being completely

***Figure 2**: Edgeworth box illustration of a patent policy*



driven out of the market. Since γ is no longer a variable that can be chosen freely by firm 2, it is clear that price competition under this policy framework will be softer than what would be the case if γ could have been chosen freely, this meaning that prices can be reasonably expected to remain higher than what would prevail in a more intense price competition scenario. This implies that a corresponding loss in consumer surplus can be expected, although this loss will most certainly be less than what would be the case under a monopoly.

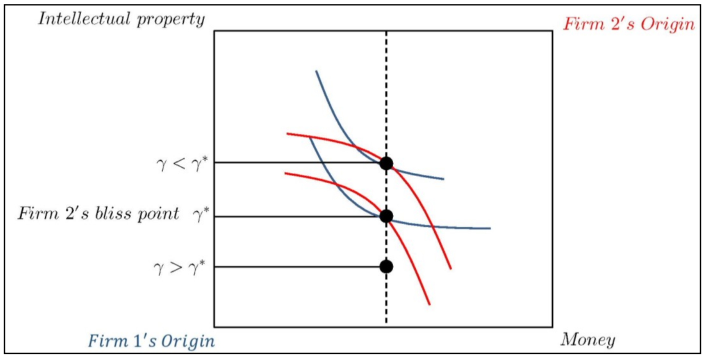
Thus, the policymaker is able to achieve the objective of maximising innovation in the market while at the same time preventing the establishment of firm 1 as a monopolist and the corresponding loss of consumer surplus that a single seller in the market would imply.

4.2. State Restriction On The Coefficient Of Imitation (γ)

Another possible policy could be a restriction on the degree of imitation to a certain degree say, 0.6 after which the imitator cannot imitate legally. Under such a policy there will be no exchange of money between the firms which means that the profits will not be internalised by firm 1. The second issue is that it is inherently subjective to decide on the legal limit of the degree of imitation. With reference to Figure 3, if the policymaker manages to choose the legal limit of γ at γ^*, then firm 2 will optimally decide to remain at this point. However, since firm 1 receives no compensation for this imitation, it cannot internalise the positive spillover effect and one can imagine firm 1's indifference curve to cross firm 2's indifference curve at this point. Similarly, for any arbitrary $\gamma < \gamma^*$ since firm 2's profit will be at the increasing portion of its profit curve in Figure 1, firm 2 will decide to operate at any such arbitrary point. However, once again firm 1 receives no compensation and its indifference curve crosses that of firm 2 at this point. For any arbitrary $\gamma > \gamma^*$, γ becomes a 'bad' for firm 2 and it will never operate at any such point.

Thus, given that firm 1 receives no compensation for giving up the subjectively set level of γ by the policymaker, any choice of γ by the policymaker will lead to a less than optimal 'production' of innovation due to the inability of firm 1 in internalising the marginal external benefit.

***Figure 3**: Imitation restriction policy*



A possible remedy in this policy framework could be that of a subsidy (or a tax cut which will have the same effect) to firm 1 for giving up the level of γ set by the policymaker, this would achieve the twin goal of maximising innovation and fostering more competition in the market. Such a policy, however, has its own problems —

1. The state will need to carry out a cost-benefit analysis before handing out subsidies, i.e., the state will need to accurately measure the externality created by the act of innovation by firm 1 so as to subsidise firm 1 accordingly. Since measuring externalities is virtually impossible, the choice of the subsidy amount will become random. Thus, in trying to solve the underproduction of innovation, the state might very well end up incurring a transfer expenditure from its budget without actually solving the problem of underproduction of innovation.

2. When analysing any policy which essentially 'rewards' an organisation (or firm 1, in our model), one must keep in mind the issue of vested political interests. Given that political interests are not an absent phenomenon in any political system around the world, such a policy might be used inappropriately in order to benefit those who the politicians favour.

Therefore, the model suggests that a patent policy aimed at promoting more innovation in the market will lead to a pareto optimal outcome and the socially optimal level of production of innovation. However, a policy that aims to foster competition by legally limiting the degree of imitation will never lead to a pareto optimal outcome and any attempt to mitigate such a policy is very much likely to create its own problems.

5. GENERALISATION TO 'n' FIRMS

In this section, we try to find an intuitive explanation as to what happens if the model is extended say, 'n' number of firms. Firstly, we analyse the situation when there is only one innovator and the rest of the firms are imitators. This analysis can be carried out by taking only one innovator because we want to analyse the effect of a singular innovative activity on all the firms. Different innovation will have analogous effects so taking many innovating firms does not give any additional information.

Assuming all the imitating firms are symmetric such that they have the same linear cost of imitation (F), then our two-firm analysis is extended analogously to the n firm situation without loss of generality.

This happens because the collection of all imitators can be taken as one big imitator as they will choose an equal coefficient of imitation (γ) as well as price, given their symmetric nature. However, if we assume that the linear cost of imitation is different for each imitating firm then this has interesting implications for the price and range of products available in the market. It is intuitively obvious that higher the cost of imitation, the lesser the incentive to imitate and lower the gamma. If we look at how the difference between the prices acts depending upon gamma, then we find that —

More closely a firm imitates the innovator, the lesser is the difference in the price between the two.

If we arrange the firms in increasing order of cost of imitation ($F_1, F_2, \dots, F_{n-1}$), then the firm with the lowest cost of imitation will have the closest product to the innovator and the closest price level. On the other hand, the firms with the highest cost of imitation will be on the other end of the spectrum. Thus, there will be a range of similar products with different price levels in the market that is caused by different imitation costs. Given the various restrictions of the above model, we were only able to provide an intuitive explanation for the n firm case. A more comprehensive model for the n firm case may provide much better insight into this area.

6. CONCLUSION

Our model gives some real insight despite being quite simplistic. We have seen that it is possible that innovation undertaken by one firm can have a positive spillover effect for the other firm as well as gaining from the investment itself. Our model also outlines the basis on which decisions such as how much to invest and how much to imitate are undertaken in the market and how these decisions are dependent upon the various characteristics of the market. Thus, an industry which is less elastic, say, a market where the products are inherently very expensive, i.e. a market for a luxury good or the market for a necessary good, the incentive to innovate is higher, this is captured by the variable 'b', this is because it offers the opportunity to charge a higher price for your innovated product without much loss of market share. Also, some of the markets are more responsive to new products or a new variety of products that it gives a higher incentive to innovate, this is captured by the parameter 'c' in our model. If we take the case of basic agricultural products, the lack of much profitability with larger investment i.e. 'c', may have characterised the lack of much innovation. Finally, the cost of imitation plays an important role in determining the decisions so a market where imitation is relatively cheap often the markets where innovation is less. This can also be extended to markets where protection is less. This is why markets such as the market of pharmaceuticals, where copying is relatively costless but involves a high fixed cost in innovation, is often one of the most protected markets in the world.

MATHEMATICAL APPENDICES

APPENDIX 1:

The base demand curves for the innovator (firm 1) and the imitator (firm 2) are as follows:

$$q_1 = \frac{A}{2} - \frac{(p_1 - p_2)b}{(1-\gamma)} + c(1-\gamma)I \qquad (A1.1)$$

$$q_2 = \frac{A}{2} + \frac{(p_1 - p_2)b}{(1-\gamma)} - c(1-\gamma)I \qquad (A1.2)$$

The corresponding cost structures for the firms are given by:

1. Short run costs

$$C_1 = \begin{cases} I\,, q_1 = 0 \\ I + mq_1,\ q_1 > 0 \end{cases} \qquad (A1.3)$$

$$C_2 = \begin{cases} 0\,, q_2 = 0 \\ mq_2,\ q_2 > 0 \end{cases} \qquad (A1.4)$$

2. Long run costs

$$C_1 = \begin{cases} I\,, q_1 = 0 \\ I + mq_1,\ q_1 > 0 \end{cases} \qquad (A1.3)$$

$$C_2 = \begin{cases} F\gamma\,, q_2 = 0 \\ F\gamma + mq_2,\ q_2 > 0 \end{cases} \quad ;\ \gamma \in [0,1] \qquad (A1.5)$$

Short run equilibrium:

Taking $\gamma = 0$ in equations (A1.1) and (A1.2) and given the short run cost structure equations (A1.3) and (A1.4), the profit functions for both the firms are given by:

$$\pi_1(p_1,p_2) = p_1\left(\frac{A}{2} - (p_1 - p_2)b + cI\right) - I - m\left(\frac{A}{2} - (p_1 - p_2)b + cI\right) \qquad (A1.6)$$

$$\pi_2(p_1,p_2) = p_2\left(\frac{A}{2} + (p_1 - p_2)b - cI\right) - m\left(\frac{A}{2} + (p_1 - p_2)b - cI\right) \qquad (A1.7)$$

Differentiating equation (A1.6) and (A1.7) w.r.t. p_1 and p_2 respectively and setting the derivatives equal to zero gives us the Bertrand reaction functions:

$$p_1 = \frac{A}{4b} + \frac{p_2}{2} + \frac{cI}{2b} + \frac{m}{2} \qquad (A1.8)$$

$$p_2 = \frac{A}{4b} + \frac{p_1}{2} - \frac{cI}{2b} + \frac{m}{2} \qquad (A1.9)$$

Substituting equation (A1.9) in equation (A1.8) gives us the Bertrand equilibrium price of firm 1 (p_1^*), which is then substituted back in equation (A1.9) to arrive at the Bertrand equilibrium price of firm 2 (p_2^*). The final equilibrium prices and profits are found as:

$$p_1^* = \frac{6mb + 3A + 2cI}{6b} \qquad (A1.10)$$

$$p_2^* = \frac{6mb + 3A - 2cI}{6b} \qquad (A1.11)$$

Putting the equilibrium prices (A1.10) and (A1.11) in (A1.6) and (A1.7) gives us the equilibrium profits —

$$\pi_1^* = \frac{4c^2I^2 + 12cAI - 36bI + 9A^2}{36b} \qquad (A1.12)$$

$$\pi_2^* = \frac{-20c^2I^2 + 24AcI + 72cbIm + 9A^2}{36b} \qquad (A1.13)$$

Long run equilibrium:

Using the same procedure as carried out above, using the long run cost structure given by equations (A1.3) and (A1.5) and the demand curves (A1.1) and (A1.2) the long run Bertrand equilibrium prices and profits are found as —

$$p_1^* = \frac{6mb + 3A(1-\gamma) + 2cI(1-\gamma)^2}{6b} \qquad (A1.14)$$

$$p_2^* = \frac{6mb + 3A(1-\gamma) - 2cI(1-\gamma)^2}{6b} \qquad (A1.15)$$

$$\pi_1^* = \frac{4c^2I^2(1-\gamma)^3 + 12cAcI(1-\gamma)^2 + 9A^2(1-\gamma) - 36bI}{36b} \qquad (A1.16)$$

$$\pi_2^* = \frac{-20c^2I^2(1-\gamma)^3 + 24AcI(1-\gamma)^2 + 72cbIm(1-\gamma) + 9A^2(1-\gamma)}{36b} - F\gamma \qquad (A1.17)$$

APPENDIX 2:

Partial derivative of (A1.16) w.r.t. c gives us:

$$\frac{\partial \pi_1^*}{\partial c} = \frac{I(1-\gamma)^2\,[3A + 2cI(1-\gamma)]}{25b} \quad > 0 \qquad (A2.1)$$

Since $\gamma \in [0,1] \rightarrow (1-\gamma) \in [0,1]$, and parameters c, I, b and A are all assumed to be positive, both the numerator and the denominator will be positive. Thus, (A2.1) will come out to be positive.

Hence, the effect of an increase in c on equilibrium profit of firm 1 is unambiguously positive.

Partial derivative of (A1.16) w.r.t. b gives us:

$$\frac{\partial \pi_1^*}{\partial b} = \frac{-4c^2I^2(1-\gamma)^3 - 12AcI(1-\gamma)^2 - 9A^2(1-\gamma)}{36b^2} \quad < 0 \qquad (A2.2)$$

Since $\gamma \in [0,1] \rightarrow (1-\gamma) \in [0,1]$, and parameters c, I, b and A are all assumed to be positive, the numerator will come out to be negative and the denominator will be positive. Thus, (A2.2) will come out to be negative.

Hence, the effect of an increase in b on equilibrium profit of firm 1 is unambiguously negative.

Partial derivative of (A1.16) w.r.t. γ gives us:

$$\frac{\partial \pi_1^*}{\partial \gamma} = \frac{-4c^2I^2(1-\gamma)^2 - 8AcI(1-\gamma) - 3A^2}{12b} < 0 \qquad (A2.3)$$

Since $\gamma \in [0,1] \rightarrow (1-\gamma) \in [0,1]$, and parameters c, I, b and A are all assumed to be positive, the numerator will be negative while the denominator will be positive. Thus, (A2.3) will come out to be negative.

Hence, the effect of an increase in γ on equilibrium profit of firm 1 is unambiguously negative.

Partial derivative of (A1.17) w.r.t. γ gives us:

$$\frac{\partial \pi_2^*}{\partial \gamma} = \frac{20c^2I^2(1-\gamma)}{12b} - \left[\frac{16AcI(1-\gamma) + 24cbIm + 3A^2}{12b} + F\right] \qquad (A2.4)$$

Since $\gamma \in [0,1] \rightarrow (1-\gamma) \in [0,1]$, and parameters c, I, b, m and A are all assumed to be positive, the denominator will be positive while the sign of the numerator is dependent on the values that the various parameters take. Thus, (A2.4) will come out to be either positive or negative.

Hence, the effect of an increase in γ on equilibrium profit of firm 2 is ambiguous and depends on the values of the various parameters.

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