EXPLORING AND MODELLING THE CONCEPT OF BOUNDED RATIONALITY IN "THE GAME OF CHICKEN"

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Abstract

Conventionally, game theoretic models assume the economic man in all strategic decision-making situations. However, this assumption of a perfectly rational human is not always true and human rationality is actually very much bounded. That is, humans might not always act for their own benefit, given the circumstances. This paper aims to incorporate this "gap in human rationality" in the game of chicken played an infinite number of times by one player, while the opponent player every time is different. The paper finds out that with increasing numbers of games the patience level of a player falls which compels him/her to make a decision that might not be favourable if they were to choose rationally and how soon they steer away from the best possible strategy depends on their idiosyncratic patience factor.

JEL Classification: D910, D83, Z13, D15
Keywords: Bounded Rationality, Impatience, Infinitely Repeated Games

1. INTRODUCTION

Game theory models the homo economicus or the economic man in all of its strategic decision-making situations. This assumption of a perfectly rational human means that the player will always act in a manner that maximises his/her payoff at the end of the game (Rittenberg and Tregarthen 2012). The player is able to analyse the entire situation at hand and choose the strategy that fetches the best outcome.

Maurice Allais (1953) and Daniel Ellsberg (1961) discovered the Allais paradox and the Ellsberg paradox respectively, both of which paved way for the development of Behavioural Game Theory. The paradoxes suggest that the decisions that the players make in a game are not consistent with the predictions made by the expected utility theory. Behavioural game theory adds to the analysis of emotions, mistakes, limited foresight, and learning. It is more concerned with what the player actually does rather than what they are expected to do (Colin F Camerer 2011).

Rather than modelling the homo economicus, whose main aim is to selfishly maximise a player's own utility, behavioural game theory uses experiments as a tool to determine a player's actual behaviour in a game (that might differ from that of the homo economicus) (Bonau Sarah 2017).

The results derived from such experiments differ from those suggested by theory as what the theoretic model often fails to incorporate is the "Bounded Rationality" of the players. The term was introduced by Herbert Simon (Simon 1957b: 198; see also Klaes and Sent 2005) to replace the assumption of perfect rationality possessed by homo economicus with such a concept of rationality that is more suited to agents with a limited reasoning ability (as often seen in real-life situations) (Wheeler 2018).

Results of experiments suggest that players often engage in altruistic cooperation or altruistic punishment (Batson et al), and show some degree of unequal aversion instead of acting purely as a homo economicus (Vailati 2016) (Lucas et al 2013). Therefore, by trying to understand the inner workings of an individual's information processing and decision-making abilities, the behavioural game theory attempts to derive results that are much similar to the real-world scenario (Bonau Sarah 2017).

In this paper, the author attempts to incorporate impatience and bounded rationality to analyse a modified version of the hawk-dove game to give a

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plausible explanation for highway accidents (other than negligent driving). The main focus of this paper shall not be the modified model but the dynamic nature of human impatience explained through the model in an attempt to give a plausible explanation for a player losing his/her rationality as the game drags on for N periods.

2. LITERATURE REVIEW

The hawk–dove game is a game-theoretic situation in which two animals (fighting over prey) can either be aggressive or passive. The ideal situation for any player is when the opponent is passive and they get to be aggressive, as opposed to the situation in which the opponent is aggressive and they have to be passive to avoid confrontation (which is the worst case). A modern adaptation of this game is the game of chicken which remains exactly the same in its modelling, outcomes, and Nash equilibria but the story presented is a bit different. In this modern adaptation, two players are driving, and approach each other with great speed. Both the players have the option to either swerve (passive) or drive straight (aggressive). Both prefer to be aggressive rather than passive but both cannot be aggressive simultaneously to prevent crashing into each other. However, both the players try to avoid yielding for the sole reason that they succumb to their pride and do not wish to look like a "chicken", that is, submit to their opponent's aggression. When one of the players' yields, the crash is avoided and the game concludes.

In this paper, the author wishes to model a situation similar to the game of chicken with identical preferences and Nash equilibria but with a slight modification, instead of both players being in motion and moving towards each other, one of the players (player 1) is stationary on one side of the road. This player is about to make a U-turn and thus some of player 1’s car’s body is covering the road, but there is enough space for player 2 (the approaching driver) to pass. (Both players are currently in the same lane and facing the same direction).

To make the model simpler, the author has assumed this road to be two-laned, with each lane designated to a particular direction and that player 2 cannot overtake player 1 by crossing the lane boundary once player 1’s car is perpendicular to the line of the road (as happens when a driver is making a U-turn). The latter assumption is a realistic one keeping in mind the possibility of oncoming traffic for the other lane. It is also assumed that the road is infinitely long. This shall play out as a sequential game with player 2 moving after player 1, and due to the assumption of an infinitely long road, focusing on player 2 for now, player 2 will have to play the same game N number of times, each time with a new player 1 (with preferences of the new player1 being the same as that of the previous one).

The above shall form the basis of the model about to be discussed. However, the author wishes to further analyse the dynamic nature of rationality in player 2 (as they are playing the same game again and again) and see what effect this dynamic nature has on the end result of this game, that is, after the Nth game.

After analysing the player's choices in a sequential game, it was concluded that in a high-stakes situation, rather than being based on rationality, decisions were based on previous outcomes (Post et al 2008). Players facing poor outcomes in succession tend to become less risk-averse. Therefore, players having exceptionally bad outcomes tend to have a higher probability gamble and continue playing rather than the average players.

Whether lucky or not, it was found that players were willing to turn down the opportunity of over a hundred per cent of the expected value of their case to continue playing. This illustrates a shift from risk-averse to a risk-seeking attitude.

This newly developed affinity towards risk in contestants who have previously been unlucky can be explained by the break-even effect. The break-even effect roughly states that a player is likely to gamble more and take risky decisions in order to win back the utility/ money/ payoff lost.

On the contrary, lucky contestants, who have had a winning streak, also tend to show an affinity towards risk and that can be attributed to the house-money effect, which suggests that players in a winning streak have a higher probability of making risky decisions because their perception that the money/utility/payoffs they are gambling with are not their own. The main result that can be derived from this analysis is that incentives drive rational choice when players make a series of decisions.

Furthermore, rational choice is also influenced by the beliefs about other people in a decision-making game. These beliefs about other players may lead to differences being observed between experimental results and utility-maximising decisions. Costa-Gomez (2008) conducted an experiment where participants were questioned, prior to the game, about their beliefs of the opponent's actions. Only 35% of the participants complied with the Nash Equilibrium. Further, only 15% of the participants stated beliefs that their opponents would choose the traditional game theory equilibrium. This meant that the participants perceived their opponents to be less rational than they actually were. This result suggests that participants refrain from choosing the utility-maximizing action and expect the same from their opponents.

3. MODEL

Consider the following situation: two drivers are situated on the road, both moving in the same direction. The driver ahead (player 1) has now stopped on the side of the road and is initiating a U-turn (i.e., stationary at present), his/her car being almost perpendicular to the line of the road (such that if the driver chooses to make the U-turn, his/her vehicle will become perpendicular to the line of the road at one point in time during which the following driver will not be able to cross driver 1). The second driver (player 2) is still in motion and is approaching the 1st driver. (See figures A.1, A.2, A.3, and A.4 in Appendix A for visual representation).

Both the drivers have 2 strategies, either to "move and pass the other driver" or "not move to let the other driver pass by".

Just as in the hawk dove scenario, both drivers are selfish and prefer to move than not move. Further, if both choose to move, there will either be an accident as in the game of chicken (here as driver 1 will become perpendicular to the road and driver 2 will collide with the side of driver 1's car) or both will be forced to halt thus restarting the game (when driver 2 comes very close to driver 1 as driver 1 starts taking the U-turn, both stop as an accident is foreseeable).

Accordingly, we have the following preferences of each player over the possible outcomes, where strategies are –

M = move
NM = not move

The order of preference and the associated utility levels of the payoffs is given by–

$$\underset{(3)}{U_1(M,NM)} > \underset{(2)}{U_1(NM,NM)} > \underset{(1)}{U_1(NM,M)} > \underset{(0)}{U_1(M,M)}$$

For player (driver) 1 and,

$$\underset{(3)}{U_2(NM,M)} > \underset{(2)}{U_2(NM,NM)} > \underset{(1)}{U_2(M,NM)} > \underset{(0)}{U_2(M,M)}$$

For player (driver) 2.

The payoff matrix thus formed is as follows:

Table 1: *Payoff Matrix*

	Player 2	
Player 1	**NM**	**M**
NM	(2,2)	(1,3)
M	(3,1)	(0,0)

Source: Author's elaboration

Therefore, as is the case with the hawk-dove game, we have two *pure-strategy* Nash equilibria. There is no dominant strategy in this game, but note that there are some differences between the chicken game and the aforementioned modified game as the author will elaborate shortly.

It should be realised that since the 2 players are in their respective cars, there is no way for the two to coordinate and play (NM, NM). Since the cars are equipped with horns and blinkers, each player can only signal to the other player about their excessive willingness to move and pass, and not about their willingness to stop.

The intensity of such indication determines the probability with which a player will choose to move or the threat that a player is giving in the case of a sequential game which we will mainly focus on.

4. ANALYSIS OF THE SEQUENTIAL GAME

This game when played sequentially becomes a lot more simplified as compared to a conventional sequential game in terms of the variety of threats available with player 2 (second mover).

Since player 2 is approaching player 1 with a certain speed, they are able to signal player 1 about their urgency via the means of horn/blinkers. By signalling to player 1 about his urgency, player 2 is essentially making a threat, the credibility of which is known to them but player 1 has to determine it. (Note that in the context of the model, signalling means making one's urgency known to the other player.)

Had we analysed the situation just like in a conventional sequential game, the subgame perfect Nash equilibrium would have been [M, (NM/M, M/NM)]. It means that player 2 would've lost the game unless player 1 had a change of heart and decided not to move.

However, the threats here, as mentioned before, do not actually have the conventional variety due to physical restrictions. Since both the players are in their respective cars, player 2 cannot give a threat in which he repeats player 1's actions or does the opposite. Due to the physical constraints, player 2 can only give one threat, that is, they will choose to move irrespective of what player 1 chooses. If player 2 does not give any threat, then the situation is nothing but a simultaneous game, the reason being that neither of the drivers engages in conflict up until the point where they both have to decide simultaneously to avoid an accident, and such a game plays out just like "the chicken game". Another possibility with no signalling is when player 2 decides to halt and let player 1 pass them by. This is an unusual case and only possible if player 2 has an altruistic nature.

The scenario we wish to develop is similar to the practical situation drivers face on a highway.

Assuming the road to be of an infinite length, player 2 (the driver in motion) is moving along this road and encounters N drivers at equally distributed intervals of time (these intervals get smaller as N rises). With each new driver who enters the game as the first mover (player 1), player 2 must play a sequential game. Since player 2 can signal about their intentions of not stopping and player 1 is able to analyse them, we shall associate probabilities to each strategy.

We will model the case wherein N becomes very large, in which player 2 plays the same game a large number of times, every time with a new player (whom we assume is playing the game for the first time and both have similar preferences over the outcomes) and come to a plausible explanation as to how these accidents occur apart from the reasons associated with negligent driving by analysing the dynamic nature of the second mover's (player 2) impatience. We shall analyse the dynamic nature of player 2's beliefs/disbeliefs and their rationality as N grows larger.

Following is the payoff matrix with probabilities assigned to each strategy (like in a mixed strategy game)-

Table 2: *Payoff matrix with mixed strategies*

Player1 \ Player2	NM (p)	M (1-p)
NM (q)	2,2	1,3
M (1-q)	3,1	0,0

Source: Author's elaboration

Let 'p' be the probability with which player 2 chooses not to move (i.e., come to a halt) and 'q' be the probability with which player 1 will choose to not move.

The above table represents a mixed strategy simultaneous game. But the author has presented the above table to make the analysis easier and to keep track. The expected utility of player 2 is our main focus since they are the driver in motion and we wish to see the changes in player 2's rationality and impatience. Being in motion, in order to prevent an accident, player 2 must choose a strategy that makes them yield in front of his opponent, which he prefers the least. We speak of expected utility as the game has not yet been played N number of times, but we wish to trace the sum of expected utilities derived from each game.

For a single game, the utility which player 2 can expect is dependent upon the probability distribution of player 1's decision.

If player 2 chooses M with probability (1-p), then they should expect a payoff of '3' (as given in the table) with probability q and '0' with probability (1-q). The same can be derived for when player 2 chooses to move with probability p.

Hence, we reach a utility function given by:

$$U2_{i=1} = [\, p^*\{2q + 1^*(1\text{-}q)\} + (1\text{-}p)^*\{3q + 0^*(1\text{-}q)\}\,]$$

$$= [\, p + 3q - 2pq]$$

For two games back-to-back, the utility function will also include a discount factor as player 2 would rather have encountered such obstacles all at once than to encounter them after a fixed interval. The reason for suggesting this assumption of "one big jam being better than many small jams" is related to inertia. The driver in motion will prefer to stay in motion unless forced to stop as a cost of effort is associated with bringing a car in motion to stop, and subsequently bringing it back into motion. Similarly, for the driver at rest, the cost of effort is associated with getting a stationary car into motion. Naturally, the driver would like to pay this implicit cost only once rather than multiple times. Although this factor plays some role in the player's decision-making process, the player's decisions are mainly driven by their desire to reach their final destination. As a result, even though there are costs associated with changing the present state of motion, the drivers will still do so in order to reach their destination.

Therefore, total utility after the 2nd encounter is given by

$$U2_{i=2} = [\, p + 3q - 2pq]\ \{1 + 1/(1+\lambda)\}$$

Where $1/(1+\lambda)$ is the discount factor or in this case, the impatience factor since the driver's impatience increases as he encounters such a scenario again and again as a result of which his utility falls by some amount. $[1/(1+\lambda) < 1$ and $\lambda > 0]$

Therefore, the expression for the Nth encounter will be the sum of a GP,

$$U2_{i=N} = [\, p + 3q - 2pq]\,(1+\lambda) - [\, p + 3q - 2pq\,]/\lambda(1+\lambda)^{N-1}$$

The above expression shows a term with a negative sign. This can be thought of as the effort cost of bringing the car back into motion after having played at least one game. That is, after having lost game N-1, they would have to bring their car back into motion in order to proceed towards the Nth game, thus costing them effort which takes away some utility. As N rises, this discomfort term falls to 0, which would mean that at the end of the N-1th game, the players would have been so used to stopping that the effort cost of doing so falls to 0. This would mean that player 2 has become so used to losing the game that losing it again won't cost them anything. Going by this chain of thought, as a driver encounters more and more obstacles, they should be expected to become more and more patient, but as we know that is not the case.

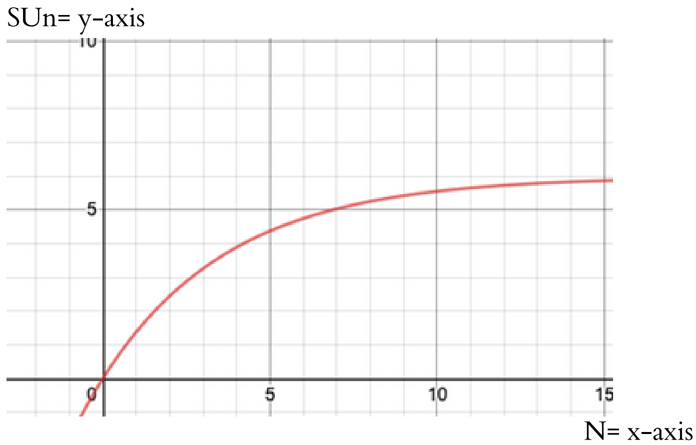
This is because there is another effect in play that might get the best of the driver, this effect is due to his/her ultimate desire to reach his/her end destination. This can be seen by taking the derivative of $U2_{i=N}$ with respect to N which is shown below

$$\delta U2_{i=N}/\delta N = [\, p + 3q - 2pq]\ \lambda^{-1}(1+\lambda)^{1-N}\,(\ln(1+\lambda)) > 0$$

$$\delta^2 U2_{i=N}/\delta N^2 = -\, [\, p + 3q - 2pq]\ \lambda^{-1}\,(\ln(1+\lambda))^2\,(1+\lambda)^{1-N} < 0$$

Therefore, as N increases, $U2_{i=N}$ increases but at a diminishing rate.

Figure 1: *As N increases,* $U2_{i=N}$ *increases but at a diminishing rate.*



Source: Author's calculation

This would mean that as N rises, due to increased impatience the total utility of the driver increases but at a diminishing rate. Thus, our assumption of "one big jam being better than many small jams" holds.

We have till now assumed that the players are rational, player 2 makes a threat and player 1 can decipher which threat is credible and which non-credible and acts accordingly to reach SGPNE.

However, by incorporating the assumption of bounded rationality we shall soon derive a realistic conclusion. What does rationality mean in this context? Rationality is not only associated with getting a higher payoff but also with the threats that the players make. A player would be considered rational only if they understand the credibility of his/her threat (something that is known to him/her) and acts accordingly. That is, if player 2 makes a non-credible threat, they understand the emptiness of his/her threat and accordingly chooses the final strategy (in this case "not move") to avoid an accident which is the worst outcome and vice versa.

Rationality would appear to be bounded when player 2 gives a non-credible threat, and despite having full knowledge of his/her threat's emptiness they expect player 1 to yield (not move). This false belief of one's threat's credibility comes from the incorporation of bounded rationality (due to rising impatience) into our model. As N increases, the rationality behind player 2's actions and belief of his threat's credibility start to fade, as a result, they begin to question the possible outcome of the next game (till now the possible outcome of the next game being player 2 must lose). It is only when N is sufficiently large that player 2's impatience reaches a level where instead of just questioning the outcome of his threat in the next game he believes (or rather wants to believe) that the outcome will be different.

Here, in our scenario where player 2 has to play the same game again and again in a single day, each time with a different player (having similar strategies and preferences). When player 2 makes a threat, he has full knowledge of it being credible or not.

Here we will make a distinction between giving a threat and acting on a threat. With the assumption of rationality, if player 2 gives a non-credible threat, then he is expected to act on it as though it really were non-credible, i.e., "chicken out" at the last moment if player 1 chooses not to respond to his/her empty threat. As a result, player 2 will lose every game up to the point where the assumption of his rationality gets violated.

Consider the following chain of arguments when we incorporate bounded rationality in the present scenario. Player 2, who has played this game N number of times, has started to grow more and more impatient. Taking derivatives tells us that as N increases, his utility increases but at a diminishing rate. His utility increases because he has to play the game again as compared to just once but at a diminishing rate because as N rises his impatience rises as well.

In the early games, the value of $p > ½$. That is, player 2 is more likely to make an empty threat and yield at the beginning (not move), showing that if N is small, defeat is more likely to be accepted. As N rises, p approaches 0, this is because the impatience is making the driver believe his non-credible threat to be credible and making him act upon them. q remains constant throughout as we are focusing only on player 2 and q of each new player 1 must be the same as each new player analyses the situation rationally for the first and last time as per our assumptions. This would mean that as N grows larger, defeat for player 2 becomes harder to accept. This can be seen from the following equations.
For some constant value of utility (U)

$$[\, p + 3q - 2pq \,](1+\lambda) - [\, p + 3q - 2pq \,]/(1+\lambda)^{N-1} = \mathbf{U}$$

$$\Rightarrow p = [\, (\mathbf{U}(1+\lambda)^{N-1}/(1+\lambda)^{N}-1) - 3q \,]\,[1/(1-2q)]$$

$$\Rightarrow dp/dN = \{[\mathbf{U}(1+\lambda)^{N-1}\ln(1+\lambda)]\,((1+\lambda)^{N}-1) - (1+\lambda)^{N}\mathbf{U}(1+\lambda)^{N-1}\ln(1+\lambda)\} \,/\, ((1+\lambda)^{N}-1)^{2}$$

$$\Rightarrow dp/dN = -(\mathbf{U}(1+\lambda)^{N-1}\ln(1+\lambda))/\, ((1+\lambda)^{N}-1)^{2} < 0$$

5. SUMMARY

We observe that as N grows larger, p tends to 0. For a large enough N, the driver would have grown so impatient that he would rather violate his homo-economicus nature and never choose to stop and an accident in such a case is inevitable.

In an ideal scenario where the driver manages to never lose rationality, due to the diminishing nature of his utility function, his utility at the end of $N = \infty$ would converge to a singular value given by

$$U_\infty = [\,p + 3q - 2pq\,](1+\lambda) / \lambda \quad \text{(from sum of infinite GP)}$$

For utility to take the above value, player 2 must stay true to his/her homo-economicus nature to the very end, and by extension; this means they will have to lose each game before they reach their destination.

For player 2 to violate his/her rationality assumption in the Nth game,

$$U2_\infty < U2_{i=N}$$

(That is, the utility derived from only playing N games must be higher than playing ∞ games.)

$$\Rightarrow [p+3q-2pq](1+\lambda)/\lambda < [\,p+3q-2pq\,](1+\lambda) - [p+3q-2pq]/(1+\lambda)^{N-1}$$

$$\Rightarrow 0 < [\lambda(1+\lambda)^N - \lambda] / (1+\lambda)^N$$

(Since λ is assumed to be positive, and $1/(1+\lambda) < 1$)

The above inequality is only possible when both terms are positive

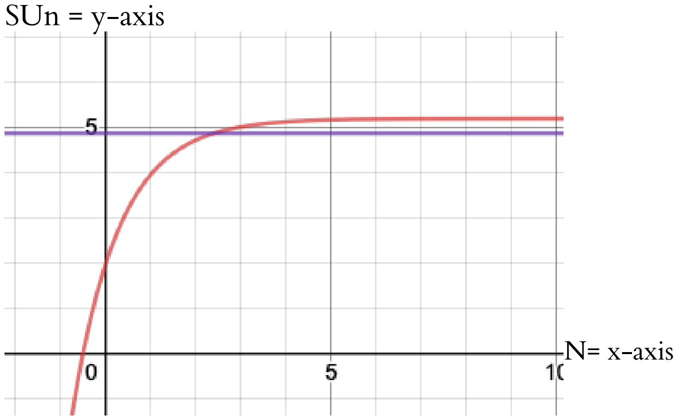
$$\Rightarrow (1+\lambda)^N > 1$$

$$\Rightarrow \lambda > 0$$

Therefore, for any value of $\lambda > 0$, $U2_\infty < U2_{i=N}$ which suggests that as N increases, an accident is bound to happen, the chances of which depend on the particular individual in the role of the second mover. The sooner they violate their homo-economicus nature (which can be attributed to their idiosyncrasies, in this case, the value of λ).

The higher the value of λ, the sooner the accident occurs.
(See graph below for reference)

Figure 2: *Higher the value of λ, the sooner the accident occurs*



Source: Author's calculation

As λ takes a higher value, the red-coloured curve moves upwards, thus intersecting the purple-coloured curve $U2_\infty$ earlier on the X-axis, and therefore, the value of N (accident occurring at the Nth game) will be smaller.

6. CONCLUSION

By means of the aforementioned model, the author has attempted to give a plausible explanation of the practical outcomes (as opposed to the ones derived from theory) by addressing the dynamic nature of human patience and bounded rationality and the process through which a player might reach such outcomes. In this case, perfect rationality would suggest accidents due to “driver impatience” may never happen, but as explained by this paper, in the real world players are not homoeconomicus, and neither do they always maximise their outcome and how an individual’s idiosyncrasies play a role in the determination of “breaking point”. Their trade-off options change with a change in their state of mind (along with other changes in their surroundings) and the “accident” in question is no more a matter of "if" but "when". It should also be noted that the model does not serve as an end, rather as a means to an end. By virtue of repetition of encounters, the model attempts to incorporate the human nature of the players due to which they tend to violate the assumption of perfect rationality and deviate from ideal behaviour.

APPENDIX A

Figure A.1

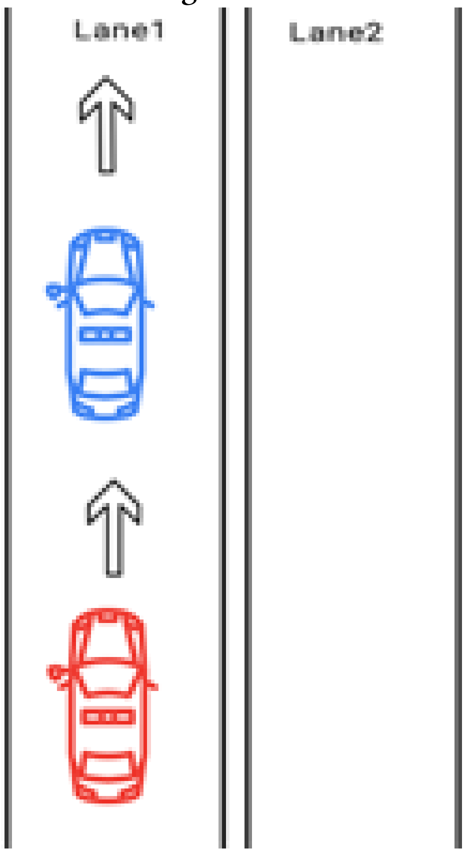


Figure A.2

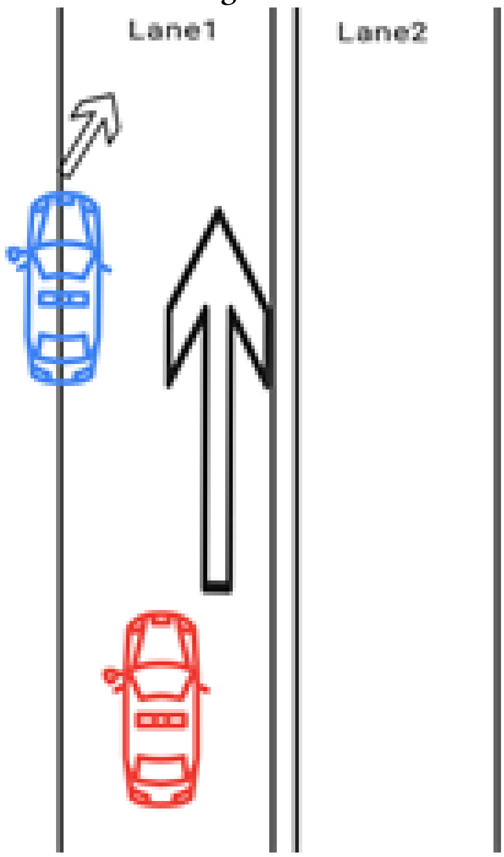


Figure A.3

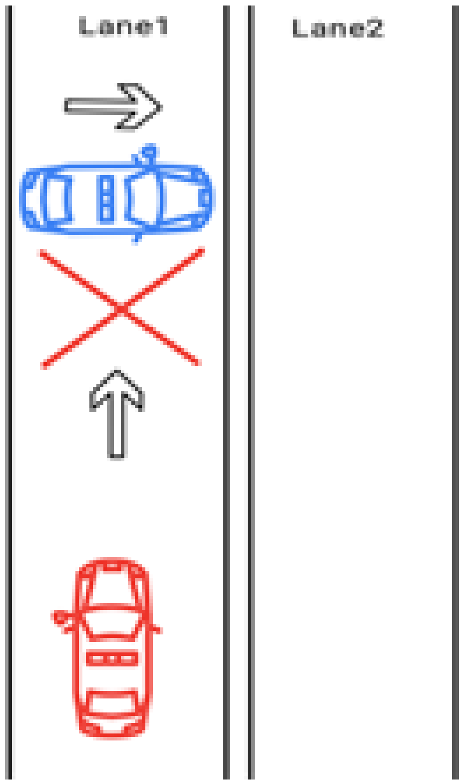
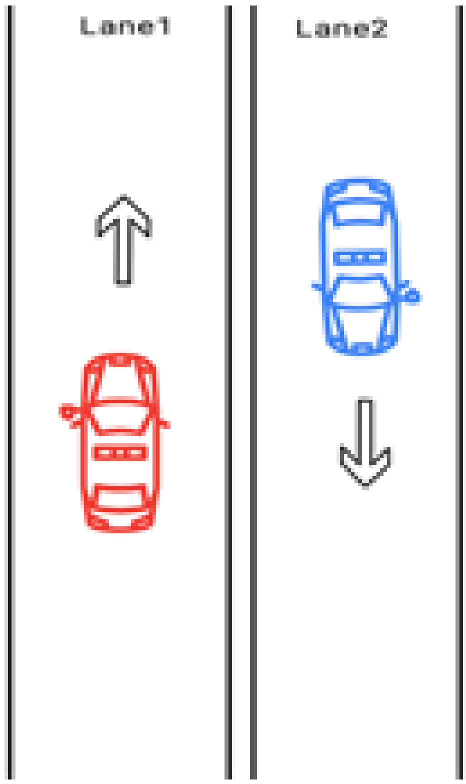


Figure A.4



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